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1975 J. Phys. A: Math. Gen. 8 L117

(http://iopscience.iop.org/0305-4470/8/10/008)

J. Phys. A: Math. Gen., Vol. 8, No. 10, 1975. Printed in Great Britain. © 1975

LETTER TO THE EDITOR

Supercritical fields and bald black holes†

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Received 15 July 1975

Abstract. The instability of a many-fermion ground state against particle–hole excitations is reviewed and the existence of supercritical electromagnetic and strong interaction fields is briefly discussed. The nature of associated phase changes and in particular the change in conservation laws which accompanies the phase changes is outlined. Finally, the supercritical gravitational field is considered and weight given to the argument that 'black holes have no hair'.

We wish to point out that the fact that 'black holes have no hair' (Misner *et al* 1973), ie a black hole is completely specified by its mass, charge, and angular momentum, and has no memory of any other quantum numbers, parity, strangeness etc which went into its composition, is simply another example of loss of symmetry during a phase transition similar to that experienced in a solid in going from a paramagnetic state to a ferromagnetic state.

To make our point we shall briefly present some very old and well known arguments and then describe some very familiar examples of phase transitions before considering the gravitational case.

In the Landau theory of an interacting many-fermion system we are introduced to the notion of quasiparticles (Brown 1972). These are the bare fermions of the system dressed by their interactions with their neighbours in the many-fermion system. There is a one-to-one correspondence between the quasiparticles and the bare fermions. Various symmetries in the system may lead to irregularities in the density of single quasiparticle states, ie band theory of solids, shell effects in atoms and nuclei etc. The interacting ground state of the system is a Fermi sea of quasiparticles corresponding to the filling of the lowest single quasiparticle states. This state has particular simplicity when it is non-degenerate as for example in the case of an insulator or a closed-shell atom or nucleus. In these cases there is a particular stability against excitation and the idea of a critical field being required before excitation can occur becomes attractive. This idea of a gap at the Fermi surface which has to be overcome before excitation can take place however can be self-generated by the correlations between the fermions during the dressing process, eg the superconducting gap. Various many-body techniques like Hartree–Fock theory are approximate prescriptions for describing the nature of the quasiparticles and almost invariably they lead to gaps at the Fermi surface. The ground state Fermi sea is then unique and corresponds to a particular energy and

† Work supported in part by USERDA Grant No. AT(11-1) 3001.

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density distribution. Excitations of the system, eg density fluctuations, are described by particle–hole configurations built on this Fermi sea and one talks about the polarizability of the system.

We shall now discuss a concrete example taken from nuclear physics (Irvine 1972) to illustrate the nature of a simple phase transition. Consider an even–even nucleus. The ground state will have angular momentum and parity 0^+; for a closed-shell configuration this is obvious, far from closed shells pairing correlations lead to a superfluid ground state also having these quantum numbers. We expect to see excited vibrational states of the system which can be described by particle–hole configurations and this is indeed the case; for closed-shell nuclei we have a low-lying 3^- octupole vibration (the quantum numbers are dictated by the alternating parity of nuclear shells) while for superfluid vibrations we have a quadrupole 2^+ vibrational state—examples are given in figure 1. The particle–hole state has the quantum numbers of a boson and it is usual

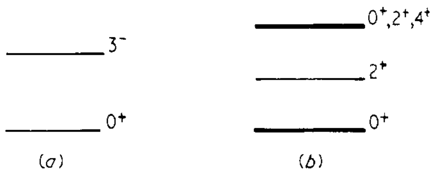

Figure 1. (*a*) Typical excitation spectrum of a closed-shell nucleus, eg ^{16}O or ^{208}Pb. (*b*) Typical vibrational spectrum of a non-closed-shell nucleus.

to discuss this vibrational spectrum in terms of phonons just as one does for density fluctuations in other systems. The energy spectrum of the phonons is governed by the density of states at the Fermi surface, ie the single quasifermion spectrum and the strength of the particle–hole interactions. If we had some mechanism for cranking up the density of states the excited vibrational state would come down and down in energy until it became degenerate with the ground state (figure 2) at which point the density fluctuations would stabilize and there would be a phase transition from a spherical to a deformed ground state configuration. Note that there is a loss of symmetry in the phase transition. Effectively this can be achieved by considering a series of nuclei, ie as we go away from the spherical $Z = 50$ configuration we enter the deformed rare

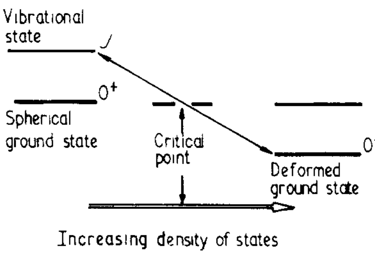

Figure 2. As the particle–hole interaction increases the vibrational state falls in energy. At the critical point there is a phase change from a spherical to a deformed ground state. Note that after the phase transition the ground state is still 0^+, since the physically observable state involves averaging over all orientations of the intrinsically deformed ground state. The uncertainty principle prohibits a knowledge of the orientation of the deformed ground state.

earth region, or as we go to masses heavier than the spherical lead nucleus we enter the deformed actinide region.

We note two features associated with the phase transition: the existence of a boson spectrum that goes to zero energy at the phase transition. Such bosons are called Goldstone bosons (Goldstone 1961). Secondly, if the Goldstone boson carries quantum numbers which are different from those of the ground state then we lose the symmetry which these quantum numbers represent, eg for both quadrupole and octupole phonons we lose sphericity since this corresponds to conservation of angular momentum. For quadrupole phonons we retain parity conservation and we have a spheroidal distortion. For octupole phonons we lose parity conservation and obtain a triaxial distortion.

Now let us consider the Dirac vacuum for electrons. In the original Dirac picture this consists of a filled sea of negative energy electron states and then a gap of 2 M_ec^2 to the allowed sea of positive energy states. The real electrons are bare electrons dressed by their interactions with the negative energy sea. All of which exactly mirrors our many-body system. The vacuum can be polarized just like the many-body system, however, because of the relatively large gap at the Fermi surface under normal circumstances vacuum polarization effects are small, eg the Lamb shift. We call the fields necessary to produce a phase change in the vacuum supercritical. Recently considerable attention has been paid to the nature of supercritical fields and the associated Goldstone bosons.

Consider a point charge $+Ze$, it will modify the electron spectrum by producing bound states which will encroach on the gap. At $Z = 1$ we recognize this as the usual hydrogen spectrum. At $Z = 1/\alpha = 137$ the binding energy of the most bound electron will equal its rest mass. However, there is no vacuum phase change associated with spontaneous electron production because of a superselection rule, lepton conservation (figure 3). If Z is increased further, eventually the most bound electron state will enter

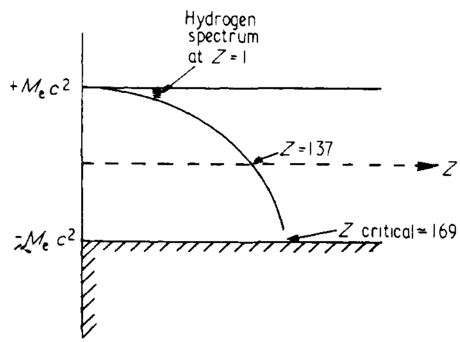

Figure 3. The electron spectrum in the presence of a point charge $+Ze$.

the negative energy continuum at which point spontaneous electron–positron pair production will occur. The Goldstone boson is in this case the positronium which is extremely unstable since the positron finds itself in a large positive charge field and will thus be strongly repelled. There is thus a divergence of the electric current provided by the fleeing of the positrons and the symmetry which is lost is charge conservation (Peiper and Greiner 1969, Muller *et al* 1972).

Consider a region of space with baryon number density B. As B is increased we shall reach a supercritical strong field at which there will be a vacuum phase change associated with the appearance of a pion condensate. The Goldstone bosons are now the pions and obviously one loses at least parity conservation (Migdal 1973, Baym and Flowers 1974).

Finally let us consider a point mass M and its effect on the spectrum of some arbitrary particle of mass m_x. The supercritical field will occur at a radius R from the mass M at which the gravitational binding energy is $2m_xc^2$, ie

$$R = GM/2c^2 \tag{1}$$

which is of course the Schwarzschild radius associated with the mass M and there will be a phase transition on the 'surface' of the black hole. What are the Goldstone bosons of the transition? We see that equation (1) is independent of m_x, ie *all* bosons simultaneously condense and thus become Goldstone bosons for the gravitational field. Hence we lose all microscopic quantum numbers in the creation of a black hole.

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